1. The algebra of the four basic operations
2. Addition and subtraction in Algebra
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The algebra of the four basic operations

MATHEMATICS

Grade 9

ALGEBRA AND GEOMETRY

Module 6

THE ALGEBRA OF THE FOUR BASIC OPERATIONS

Activity 1

To refresh understanding of conventions in algebra as applied to addition and subtraction

[LO 1.2, 1.6]

A We will first have a look at terms.

Remember, terms are separated by + or –. In each of the following, say how many terms there are:

1. a + 5

2. 2a2

3. 5a (a+1)

4. $\frac{3a-1}{a^2} + 5a$

In the next exercise you must collect like terms to simplify the expression:

1. 5a + 2a

2. 2a2 + 3a – a2

3. 3x – 6 + x + 11

4. 2a(a–1) – 2a2

B Adding expressions

Example:

Add 3x + 4 by x + 5.

(x + 5) + (3x + 4) Write, with brackets, as sum.

x + 5 + 3x + 4 Remove brackets, with care.

4x + 9 Collect like terms.

In this exercise, add the two given expressions:

1. 7a + 3 and a + 2

2. 5x – 2 and 6 – 3x

3. x + ½ and 4x – 3½

4. a2 + 2a + 6 and a – 3 + a2

5. 4a2 – a – 3 and 1 + 3a – 5a2

C Subtracting expressions

Study the following examples very carefully:

Subtract 3x – 5 from 7x + 2.

(7x + 2) – (3x – 5)

Notice that 3x – 5 comes second, after the minus.

7x + 2 – 3x + 5

The minus in front of the bracket makes a difference!

4x + 7

Collecting like terms.

Calculate 5a – 1 minus 7a + 12: (5a – 1) – (7a + 12)

5a – 1 – 7a – 12

–2a – 13

D Mixed problems

Do the following exercise (remember to simplify your answer as far as possible):

1. Add 2a – 1 to 5a + 2.

2. Find the sum of 6x + 5 and 2 – 3x.

3. What is 3a – 2a2 plus a2 – 6a?

4. (x2 + x) + (x + x2) = . . .

5. Calculate (3a – 5) – (a – 2).

6. Subtract 12a + 2 from 1 + 7a.

7. How much is 4x2 + 4x less than 6x2 – 13x?

8. How much is 4x2 + 4x more than 6x2 – 13x?

9. What is the difference between 8x + 3 and 2x +1?

Use appropriate techniques to simplify the following expressions:

1. x2 + 5x2 – 3x + 7x – 2 + 8

2. 7a2 – 12a + 2a2 – 5 + a – 3

3. (a2 – 4) + (5a + 3) + (7a2 + 4a)

4. (2x – x2) – (4x2 – 12) – (3x – 5)

5. (x2 + 5x2 – 3x) + (7x – 2 + 8)

6. 7a2 – (12a + 2a2 – 5) + a – 3

7. (a2 – 4) + 5a + 3 + (7a2 + 4a)

8. (2x – x2) – 4x2 – 12 – (3x – 5)

9. x2 + 5x2 – 3x + (7x – 2 + 8)

10. 7a2 – 12a + 2a2 – (5 + a – 3)

11. a2 – 4 + 5a + 3 + 7a2 + 4a

12. (2x – x2) – [(4x2 – 12) – (3x – 5)]

Here are the answers for the last 12 problems:

1. 6x2 + 4x + 6

2. 9a2 – 11a – 8

3. 8a2 + 9a – 1

4. – 5x2 – x + 17

5. 6x2 + 4x + 6

6. 5a2 – 11a + 2

7. 8a2 + 9a – 1

8. – 5x2 – x – 7

9. 6x2 + 4x + 6

10. 9a2 – 13a – 2

11. 8a2 + 9a – 1

12. – 5x2 + 5x + 7

Activity 2

To multiply certain polynomials by using brackets and the distributive principle

[LO 1.2, 1.6, 2.7]

A monomial has one term; a binomial has two terms; a trinomial has three terms.

A Multiplying monomials.

Brackets are often used.

Examples:

2a × 5a = 10a2

3a3 × 2a × 4a2 = 24 a6

4ab × 9a2 × (–2a) × b = –36a4b2

a × 2a × 4 × (3a2)3 = a × 2a × 4 × 3a2 × 3a2 × 3a2 = 126a8

(2ab2)3 × (a2bc)2 × (2bc)2 = (2ab2) (2ab2) (2ab2) × (a2bc) (a2bc) × (2bc) (2bc) = 32a7b10c4

Always check that your answer is in the simplest form.

Exercise:

1. (3x) (5x2)

(x3) (–2x)

(2x)2 (4)

(ax)2 (bx2) (cx2)2

B Monomial × binomial

Brackets are essential.

Examples:

5(2a + 1) means multiply 5 by 2a as well as by 1. 5 (2a + 1) = 10a + 5

Make sure that you work correctly with your signs.

4a(2a + 1) = 8a + 4a

–5a(2a + 1) = –10a2 – 5a

a2(–3a2 – 2a) = –3a4 – 2a3

–7a(2a – 3) = –14a2 + 21a

Note: We have turned an expression in factors into an expression in terms. Another way of saying the same thing is: A product expression has been turned into a sum expression.

Exercise:

1. 3x (2x + 4)

x2 (5x – 2)

–4x (x2 – 3x)

(3a + 3a2) (3a)

C Monomial × trinomial

Examples:

5a(5 + 2a – a2) = 25a + 10a2 – 5a3

– ½ (10x5 + 2a4 – 8a3) = – 5x5 – a4 +4a3

Exercise:

3x (2x2 – x + 2)

–ab2 (–bc + 3abc – a2c)

12a (¼ + 2a + ½ a2)

Also try: 4. 4x (5 – 2x + 4x2 – 3x3 + x4)

D Binomial × binomial

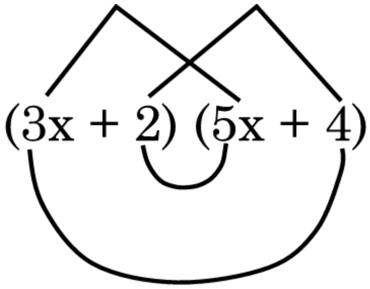
Each term of the first binomial must be multiplied by each term of the second binomial.

(3x + 2) (5x + 4) = (3x)(5x) + (3x)(4) + (2)(5x) + (2)(4) = 15x2 + 12x + 10x + 8

= 15x2 + 22x + 8

Always check that your answer has been simplified.

Here is a cat–face picture to help you remember how to multiply two binomials:



The left ear says multiply the first term of the first binomial with the first term of the second binomial.

The chin says multiply the first term of the first binomial with the second term of the second binomial.

The mouth says multiply the second term of the first binomial with the first term of the second binomial.

The right ear says multiply the second term of the first binomial with the second term of the second binomial.

There are some very important patterns in the following exercise – think about them.

Exercise:

(a + b) (c + d)

(2a – 3b) (–c + 2d)

(a2 + 2a) (b 2 –3b)

(a + b) (a + b)

(x2 + 2x) (x2 + 2x)

(3x – 1) (3x – 1)

(a + b) (a – b)

(2y + 3) (2y – 3)

(2a2 + 3b) (2a2 – 3b)

(a + 2) (a + 3)

(5x2 + 2x) (x2 – x)

(–2a + 4b) (5a – 3b)

E Binomial × polynomial

Example:

(2a + 3) (a3 – 3a2 + 2a – 3) = 2a4 – 6a3 + 4a2 – 6a + 3a3 – 9a2 + 6a – 9

= 2a4 – 3a3 – 5a2 – 9 (simplified)

Exercise:

(x2 – 3x) (x2 + 5x – 3)

(b + 1) (3b2 – b + 11)

(a – 4) (5 + 2a – b + 2c)

(–a + 2) (a + b + c – 3d)

Activity 3

To find factors of certain algebraic expressions

[LO 1.6, 2.1, 2.7]

A Understanding what factors are

This table shows the factors of some monomials

Expression	Smallest factors
42	2 × 3 × 7
6ab	2 × 3 × a × b
21a2b	3 × 7 × a × a × b

(5abc2)2	5 × a × b × c × c × 5 × a × b × c × c
–8y4	–2 × 2 × 2 × y × y × y × y
(–8y4)2	–2 × 2 × 2 × y × y × y × y × –2 × 2 × 2 × y × y × y × y

You can write the factors in any order, but if you stick to the usual order your work will be easier. Two lists of factors in the table are not in the usual order – rewrite them in order.

B Finding common factors of binomials

Take the binomial 6ab + 3ac.

The factors of 6ab are 2 × 3 × a × b and the factors of 3ac are 3 × a × c.

The factors that appear in both 6ab and 3ac are 3 and a – they are called common factors.

We can now use brackets to group the factors into the part that is common and the rest, as follows:

6ab = 3a × 2b and 3ac = 3a × c

Now we can factorise 6ab + 3ac. This is how to set it out:

6ab + 3ac = 3a (2b + c).

An expression in terms has been written as an expression in factors.

Or: A sum expression has been turned into a product expression.

Here are some more examples:

6x2 + 12x = 3x (2x + 4)

5x3 – 2x2 = x2 (5x – 2)

–4x3 + 12x2 = –4x (x2 – 3x)

9a2 + 9a3 = (3a + 3a2) (3a)

Look back at the exercise in section B of the previous activity – did you recognise the problems?

C Finding common factors of polynomials

In exactly the same way we can find the common factors of more than two terms. Here are some examples:

Examples:

6x3 – 3x2 + 6x = 3x (2x2 – x + 2)

ab3c – 3a2b3c + a3b2c = ab2c (b – 3ab + a2)

3a + 24a2 + 6a3 = 3a (1 + 8a + 2a2)

20x – 8x2 + 16x3 – 12x4 +4x5 = 4x (5 – 2x + 4x2 – 3x3 + x4)

You will notice that the terms remaining in the brackets don’t have any more common factors left. This is because they have been fully factorised. You must always take out the highest common factor from all the terms.

Exercise:

Fully factorise the following expressions by taking out the highest common factor:

12abc + 24ac

15xy – 21y

3abc + 18ab2c3

8x2y2 – 2x

2a2bc2 + 4ab2c – 7abc

12a(bc)2 – 8(abc)3 + 4(ab)2c3 – 20bc + 4a

Pair activity:

Did you notice that in each case the number of terms in the brackets after factorising was the same as the number of terms in the original expression?

Explain to your partner why you think this will always happen.

D Factorising difference of squares

In section D of the previous activity you had to multiply these three pairs of binomials:

(a + b) (a – b) ,

(2y + 3) (2y – 3) and

(2a2 + 3b) (2a2 – 3b)

Here are the solutions:

(a + b) (a – b) = a2 – b2

(2y + 3) (2y – 3) = 4y2 – 9

(2a2 + 3b) (2a2 – 3b) = 4a4 – 9b2

You will have noticed that the answers have a very special pattern: square minus square.

This is called a difference of squares and this is how it is factorised:

First–square minus second–square

= ($\sqrt{\text{first} - \text{square}}$plus $\sqrt{\text{second} - \text{square}}$) ($\sqrt{\text{first} - \text{square}}$minus $\sqrt{\text{second} - \text{square}}$)

Examples:

x2 – 25 = (x + 5) (x – 5)

4 – b2 = (2 + b) (2 – b)

9a2 – 1 = (3a + 1) (3a – 1)

YOU HAVE TO BE VERY SURE OF THE MOST COMMON SQUARES AND THEIR ROOTS.

These are a few important ones – you must add to this list.

22 = 4 32 = 9 (a2)2 = a4

(a3)2 = a6

(½)2 = ¼ 12 = 1

Exercise:

Factorise fully:

1. a2 – b 2

4y2 – 9

4a4 – 9b2

1 – x2

25 – a6

a8 – ¼

4a2b 2 – 81

0,25 – x2y6

9. 2a2 – 2b2 (take care!)

E Combining common factors with differences of squares

As you saw in the last exercise (number 9), it is essential to check for common factors first and then to factorise the bracketed polynomial if possible.

Another example:

Factorise 12ax2 – 3ay2

First recognise that there is a common factor of 3a, before saying that this can’t be a difference of squares.

12ax2 – 3ay2 = 3a (4x2 – y2) Now we recognise 4x2 – y2 as the difference of two squares.

12ax2 – 3ay2 = 3a (4x2 – y2) = 3a(2x + y)(2x – y).

Exercise:

Factorise completely:

1. ax2 – ay4

2. a3 – ab2

3. 0,5a2x – 4,5b2x

4. a5b3c – abc

F Successive differences of squares

Try factorising this binomial completely by keeping your eyes open: a4 – b4

Now do this exercise – as usual factorise as far as possible.

1. x6 – 64

2. 1 – m8

3. 3a4 – 24b8

4. x – x9

G Factorising trinomials

If you study the answers to the following four problems (they are from a previous activity), you will notice that the answers, after simplifying, sometimes have two terms, sometimes three terms and sometimes four terms. Discuss what you see happening (with a partner) and decide why they are different.

1. (a + b) (a – b) = a2 – ab + ab – b2 = a2 – b2 (when simplified)

2. (a + 2) (a + 3) = a2 + 3a + 2a + 6 = a2 + 5a + 6

3. (a + b) (a + b) = a×a +ab + ba + b×b = a2 + ab + ab + b2 = a2 + 2ab + b2 (in the simplest form)

4. (a + b) (c + d) = ac + ad + bc + bd (and this answer can not be simplified)

The answer to the type of problem represented by number 1 above is a difference of squares.

The answers to numbers 2 and 3 are trinomials, and we will now see how to factorise them

The first fact that you must always remember is that not all trinomials can be factorised.

Work backwards through problem 2:

a2 + 5a + 6 = a2 + 3a + 2a + 6 = (a + 2) (a + 3).

So that you can see clearly where the a2 came from, and the 5a and the 6.

Now try to factorise a2 + 7a + 12 = (………….) (……………) by filling in two correct binomials in the brackets.

You can check your answer by multiplying the binomials in your answer as you were taught in activity 2. Keep trying and checking your answer until you have it right. Do the same in the following three exercises:

Match up the two columns:

A. a2 – 5a – 6 1. (x + 2)(x + 3)

B. a2 – a – 6 2. (x – 2)(x + 3)

C. a2 – 5a + 6 3. (x + 1)(x – 6)

D. a2 + 7a + 6 4. (x – 2)(x – 3)

E. a2 + 5a + 6 5. (x + 1)(x + 6)

F. a2 + 5a – 6 6. (x – 1)(x + 6)

G. a2 + a – 6 7. (x + 2)(x – 3)

H. a2 – 7a + 6 8. (x – 1)(x – 6)

Now factorise the following trinomials by using the same techniques you have just learnt. The last two are more difficult than the first four!

a2 + 3a + 2

a2 + a – 12

a2 – 4a + 3

a2 – 9a + 20

a2 + ab – 12b2

2a2 – 18a + 40

Problem	**One factor correct**	**Both factors correct**	**Answer checked by multiplication**
	1	2	3
1			
2			
3			
4			
5			
6			

Finally, in groups of 3, 4 or 5, work out exactly how one should go about factorising these trinomials, and write down a strategy that will get you to the answer accurately and quickly.

Activity 4

To use factorising in simplifying fractions, and in multiplying, dividing and adding fractions

[LO 1.2, 1.6, 2.9]

A. Simplifying algebraic fractions

Two of the following four fractions can be simplified and the others can’t. Which is which?

$\frac{2+a}{2-a}$

$\frac{3(a+b)}{a+b}$

$\frac{4+x}{x+4}$

Equation:

$$\frac{a(b-c)}{2(b+c)}$$

As you have seen in the previous activity, factorising is a lot of trouble. So, why do we do it?

The following expression cannot be simplified as it is $\frac{6\mathrm{a}^2b-6\mathrm{b}}{2\mathrm{a}-2}$ because we are not allowed to cancel terms. If we can change the sum expressions into product expressions (by factorising) then we will be able to cancel the factors, and simplify.

6a2b – 6b = 6b (a2 – 1) = 6b (a + 1) (a – 1) and 2a – 2 = 2(a – 1)

So, the reason we factorise is that it allows us to simplify expressions better.

Now: $\frac{6\mathrm{a}^2b-6\mathrm{b}}{2\mathrm{a}-2} = \frac{6b(a+1)(a-1)}{2(a-1)} = \frac{3\mathrm{b}(a+1)}{1}$ = 3b(a + 1) .

It is very important that you factorise completely.

Exercise:

Factorise the numerator and denominator, cancel factors and write in the simplest form:

1 $\frac{12a+6\mathrm{b}}{2\mathrm{a}+b}$

2 $\frac{x^2-9}{x+3}$

3 $\frac{2(a+1)(a-1)}{6(a+1)^2}$

4 $\frac{5a^2-5}{5a+5}$

B. Multiplying and dividing fractions

Our normal rules for multiplication and division of fractions are still the same. Study the following examples, taking special note of the factorising and cancelling.

$$\frac{4x^3y}{6y^2} \div \frac{xy}{3x^2} \times \frac{2xy^2}{3x} = \frac{4x^3y}{6y^2} \times \frac{3x^2}{xy} \times \frac{2xy^2}{3x} = \frac{4x^4}{3}$$

$$\frac{a^2-9}{2} \times \frac{1}{4a^2-12a} = \frac{(a+3)(a-3)}{2} \times \frac{1}{4a(a-3)} = \frac{(a+3)}{8a}$$

$$\frac{3a+6}{5} \div \frac{a^2-4}{10} = \frac{3a+6}{5} \times \frac{10}{a^2-4} = \frac{3(a+2)}{5} \times \frac{10}{(a+2)(a-2)} = \frac{6}{a-2}$$

Exercise:

Simplify:

1. $\frac{2ab^2}{b^3c} \times \frac{9ac^2}{4b} \div \frac{3ac}{2b^2}$

2. $\frac{2(a+1)(a-2)^2}{(a-2)^3(a+3)} \times \frac{9(a+1)(a+3)^2}{4(a-2)} \div \frac{3(a+1)(a+3)}{2(a-2)^2}$

3. $\frac{4a^2+8a}{2b+4} \times \frac{3\left(b^2+2\right)}{3a^2+6a}$

4. $\frac{x^2-1}{5x-5} \div \frac{(x+1)^2}{15x+15}$

5. $\frac{7x}{3xy} \div \frac{3x+6}{5x^2y} \div \frac{5x-10}{3x^2-12}$

6. $\frac{\frac{5x^2+5x}{x^2-x}}{\frac{5x+5}{x^2-1}}$ (here we have a fraction divided by a fraction – first rewrite it like number 4)

C. Adding fractions

You already know quite well that adding and subtracting fractions is a lot more difficult than multiplying and dividing them. The reason is that we can only add and subtract the same kind of fractions, namely fractions with identical denominators. If the denominators are different, find the lowest common multiple of the denominators (LCD), rewrite all the terms with this as denominator, and then simplify by gathering like terms together. Finally, the answer has to be simplified by cancelling factors occurring in both numerator and denominator. Here are some examples – all the steps have been shown:

Simplify:

1. $\frac{5abx}{2cx} + \frac{4ac}{3x} + \frac{cx}{2a}$ (LCD = 6acx)

$$\left(\frac{5abx}{2cx} \times \frac{3a}{3a}\right) + \left(\frac{4ac}{3x} \times \frac{2ac}{2ac}\right) + \left(\frac{cx}{2a} \times \frac{3cx}{3cx}\right) = \frac{15a^2bx}{6acx} + \frac{8a^2c^2}{6acx} + \frac{3c^2x^2}{6acx} = \frac{15a^2bx+8a^2c^2+3c^2x^2}{6acx}$$

2. $\frac{a+b}{2} + \frac{b+c}{3} - \frac{a+c}{6}$ (LCD = 6) Take careful note of how the signs are handled below!

$$\frac{3(a+b)}{6} + \frac{2(b+c)}{6} - \frac{a+c}{6} = \frac{3(a+b)+2(b+c)-(a+c)}{6} = \frac{3a+3b+2b+2c-a-c}{6} = \frac{2a+5b+c}{6}$$

3. $\frac{a+3}{a^2-4} + \frac{1}{3a+6} + \frac{2}{5a-10}$

To find Lowest Common Denominator first factorise denominators!

$$\frac{a+3}{(a+2)(a-2)} + \frac{1}{3(a+2)} + \frac{2}{5(a-2)}$$

Do you see that the LCD is 3×5×(a+2)(a–2)?

$$\left(\frac{a+3}{(a+2)(a-2)}\times\frac{15}{15}\right)+\left(\frac{1}{3(a+2)}\times\frac{5(a-2)}{5(a-2)}\right)+\left(\frac{2}{5(a-2)}\times\frac{3(a+2)}{3(a+2)}\right)$$

$$=\frac{15(a+3)+5(a-2)+6(a+2)}{15(a+2)(a-2)}=\frac{15a+45+5a-10+6a+12}{15(a+2)(a-2)}=\frac{26a+47}{15(a+2)(a-2)}$$

Exercise:

Simplify the following expressions by using factorising:

1. $\frac{a}{x^2}-\frac{a}{x}+\frac{5a}{2x}$
2. $\frac{1}{3}+\frac{2x+1}{2x}-\frac{x-1}{3x}$
3. $\frac{4a-4b}{2a^2-2b^2}-\frac{3}{2a-2b}$
4. $\frac{1}{2}(a-2)+\frac{2}{3}(a+1)-\frac{3}{4}(a-3)$

Assessment: Assess the 4 problems in the exercise above.

Here is one final trick. We could simplify $\frac{2(x-1)}{3(x+3)}\times\frac{9(x+3)}{(1-x)}$ better if (1–x) had been: (x–1).

So, we make the change we want by changing the sign of the whole binomial as well:

(1–x) = –(x–1) because –(x–1) = –x + 1, which is 1–x. Finish the problem yourself.

Assessment

Learning outcomes(LOs)

LO 1
Numbers, Operations and RelationshipsThe learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.
Assessment standards(ASs)
We know this when the learner:
1.1 describes and illustrates the historical development of number systems in a variety of historical and cultural contexts (including local);
1.2 recognises, uses and represent rational numbers (including very small numbers written in scientific notation), moving flexibly between equivalent forms in appropriate contexts;
1.3 solves problems in context, including contexts that may be used to build awareness of other Learning Areas, as well as human rights, social, economic and environmental issues such as:
1.3.1 financial (including profit and loss, budgets, accounts, loans, simple and compound interest, hire purchase, exchange rates, commission, rentals and banking);
1.3.2 measurements in Natural Sciences and Technology contexts;
1.4 solves problems that involve ratio, rate and proportion (direct and indirect);

1.5 estimates and calculates by selecting and using operations appropriate to solving problems and judging the reasonableness of results (including measurement problems that involve rational approximations of irrational numbers);	
1.6 uses a range of techniques and tools (including technology) to perform calculations efficiently and to the required degree of accuracy, including the following laws and meanings of exponents (the expectation being that learners should be able to use these laws and meanings in calculations only):	
1.6.1 x n × x m = xn + m	
1.6.2 x n x m = xn – m	
1.6.3 x 0 = 1	
1.6.4 x –n = $\frac{1}{x^n}$	
1.7 recognises, describes and uses the properties of rational numbers.	
LO 2	
Patterns, Functions and AlgebraThe learner will be able to recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills.	
We know this when the learner:	
2.1 investigates, in different ways, a variety of numeric and geometric patterns and relationships by representing and generalising them, and by explaining and justifying the rules that generate them (including patterns found in natural and cultural forms and patterns of the learner's own creation);	

2.7 uses the distributive law and manipulative skills developed in Grade 8 to:
find the product of two binomials;
factorise algebraic expressions (limited to common factors and difference of squares).
2.8 uses the laws of exponents to simplify expressions and solve equations;
2.9 uses factorisation to simplify algebraic expressions and solve equations.

Memorandum

TEST 1

1. Simplify the following expressions by collecting like terms:

1.1 3a2 + 3a2 – 6a + 3a – 4 + 1

1.2 2y2 – 1y + 2y2 – 6 + 2y – 9

1.3 8x2 – (5x + 12x2 – 1) + x – 4

1.4 (3a – a2) – [(2a2 – 11) – (5a – 3)]

2. Give the answers to the following problems in the simplest form:

2.1 Add 3x2 + 5x – 1 to x2 – 3x

2.2 Find the sum of 2a + 3b – 5 and 3 + 2b – 7a

2.3 Subtract 6a + 7 from 5a2 + 2a + 2

2.4 How much is 3a – 8b + 3 less than a + b + 2?

3. Simplify by multiplication, leaving your answer in the simplest form:

3.1 (3x2) × (2x3)

3.2 (abc) (a2c) (2b2)

3.3 abc(a2c + 2b2)

3.4 –3a(2a2 – 5a)

3.5 (a – 2b) (a + 2b)

3.6 (3 – x2) (2x2 + 5)

3.7 (x – 5y)2

3.8 (2 – b) (3a + c)

TEST 1 – Memorandum

1.1 6a2 – 3a – 3

1.2 4y2 + y – 15

1.3 – 4x2 – 4x – 3

1.4 – 3a2 + 8a + 8

2.1 4x2 + 2x – 1

2.2 – 5a + 5b – 2

2.3 5a2 – 4a – 5

2.4 – 2a + 9b – 1

3.1 6x5

3.2 2a3b3c2

3.3 a3bc2 + 2ab3c

3.4 – 6a3 + 15a2

3.5 a2 – 4b2

3.6 –2x4 + x2 + 15

3.7 x2 – 10xy + 25y2

3.8 6a + 2c – 3ab – bc

TEST 2

1. Find the Highest Common Factor of these three expressions: 6a2c2 and 2ac2 and 10ab2c3.

2. Completely factorise these expressions by finding common factors:

2.1 12a3 + 3a4

2.2 –5xy – 15x2y2 – 20y

2.3 6a2c2 – 2ac2 + 10ab2c3

3. Factorise these differences of squares completely:

3.1 a2 – 4

3.2 $\frac{1}{9}a^2 - 9b^2$

3.3 x4 – 16y4

3.4 1 – a4b4

4. Factorise these expressions as far as possible:

4.1 3x2 – 27

4.2 2a – 8ab2

4.3 a2 – 5a – 6

4.4 a2 + 7a + 6

5. Simplify the following fractions by making use of factorising:

5.1 $\frac{3a^2-3}{6a+6}$

5.2 $\frac{6x^2y-6y}{2x-2}$

5.3 $\frac{a^2-9}{2} \times \frac{1}{4a^2-12a}$

5.4 $\frac{3x+6}{5} \div \frac{x^2-4}{15}$

5.5 $\frac{abx}{2cx} + \frac{2ac}{3x} + \frac{3cx}{2a}$

5.6 $\frac{2a}{x^2} - \frac{3a}{x} + \frac{a}{2x}$

5.7 $\frac{4a-4b}{2a^2-2b^2} - \frac{3}{2a-2b}$

5.8 $\frac{2}{3}(a+2) + \frac{1}{3}(a-1) - \frac{1}{4}(a-5)$

TEST 2 – Memorandum

1. 2ac2

2.1 3a3 (4 + a2)

2.2 –5y (x + 3x2y + 4)

2.3 2ac2 (3a – 1 + 5b2c)

3.1 (a + 2) (a – 2)

3.2 $\left(\frac{1}{3}a + 3\mathrm{b}\right)\left(\frac{1}{3}a - 3\mathrm{b}\right)$

3.3 (x2 + 4y2) (x + 2y) (x – 2y)

3.4 (1 + a2b2) (1 + ab) (1 – ab)

4.1 3 (x + 3) (x – 3)

4.2 2a (1 + 4b) (1 – 4b)

4.3 (a + 1) (a – 6)

4.4 (a + 1) (a + 6)

5.1 $\frac{a-1}{2}$

5.2 3y (x + 1)

5.3 $\frac{a+3}{8\mathrm{a}}$

5.4 $\frac{9}{x-2}$

5.5 $\frac{3\mathrm{a}^2\mathrm{bx}+4\mathrm{a}^2c^2+9\mathrm{c}^2x^2}{6\mathrm{acx}}$

5.6 $\frac{4\mathrm{a}-5\mathrm{ax}}{2\mathrm{x}^2}$

5.7 $\frac{a-7\mathrm{b}}{2(a+b)(a-b)}$

5.8 $\frac{3\mathrm{a}+9}{4}$

Addition and subtraction in Algebra

MATHEMATICS

Grade 8

THE NUMBER SYSTEM

(Natural and whole numbers)

Module 4

ADDITION AND SUBTRACTION IN ALGEBRA

CLASS ASSIGNMENT1

- **Discover more and more about addition and subtraction in ALGEBRA ….**

- When we do addition and subtraction in Algebra, we can only add together or subtract like terms (terms of the same type) from one another.
- e.g. $3a + 5a = 8a$ (a is the same for both terms)
- e.g. $3a + 5a^2$ (cannot be added together, because one term is a and the othera^2 - not of the same type)
- remember: -8(+5) means: -8 x (+5) = -40(two signs must not be placed next to one another; multiply the two signs by one another)

1.Can you still do the following with integers?

1.1: -8 - 12 + 8

1.2: 7 - (+8) - (-6)

1.3: 15 - (-9) + (+7)

1.4: 2(-6) - 5(-6)

1.5: 50 - 70 + 15

2. WOW! Look at this! Quite easy….

2.1 $2a + 2a =$ (yes, they are alike; I can add)

2.2 $3a - 6y + 7a + 15y =$ (look for like terms)

$3a + 7a =$

$-6y + 15y =$

(now write the answer alongside the question)

2.3 Add the following expressions together:

2.3.1: $6a - 7b - 9c$; $-7a + 15b - 29c$

2.3.2: $-9a^2 - 16a + 17b$; $-17a^2 -40$; $-29b + 30$

3. How about subtracting? Look at the following example:Subtract 6 from 15 . How would you write this? $15 - (+6) = 15 - 6 = 9$

Explanation: (-) x (+) = (-) THEREFORE: $15 - 6$

Look at the following: Subtract $-6a + 5b$ from $16a - 3b$

It will look like this: $16a - 3b - (-6a + 5b)$

$= 16a - 3b + 6a - 5b$ (multiply (-) within the brackets)

$= 16a + 6a - 3b - 5b$

$= 22a - 8b$

- Important: Begin by deciding which expression should be written first!

3.1 Calculate each of the following:

3.1.1 Subtract the second from the first: $-7a + 3$; $6a - 9$

3.1.2 Subtract $-7a^2 - 5a + 8$ from $18a^2 - 15$

3.1.3 Reduce $-15x^2 - 7x + 20$ by $-6x^2 + 76$

HOMEWORK ASSIGNMENT1

1. Add the following expressions together: $-3p^2 - 2p - 5$ **;** $6p^2 + 8$ **;** $-15p - 28$

2. Subtract $-5p^2 - 3p$ from $8 - 7a^2$

3. Subtract $6a - 8y$ from 1

4. $3a^3 + 6a - 7a - 5 - 2(8a^3 - 4a^2 + 17a + 8) - 15a$

5. Increase $6p + 15y - 3a$ by $-13y - 18p + 34a$

6. By how much is $-8a^2 + 6a$ bigger than $15a^2 + 3a - 5$?

7. By how much is $4a^2 - 5a + 1$ smaller than $16a^2 + 3a - 7$?

8. What must be added to $5a^2 + 3a$ to get $-3a + 6$?

Assessment

Assessment of myself:		by myself:				Assessment by Teacher:							
I can…						1	2	3	4		Critical Outcomes	1	2
Identify like terms in an											Critical and creative thinking		

expression; (Lu 2.8.1; 2.8.2)														
Add like terms together; (Lu 2.8.2; 2.8.4)											Collaborating			
Subtract like terms from each other; (Lu 2.8.2; 2.8.4)											Organising en managing			
Add and subtract constant values; (Lu 2.8.4)											Processing of information			
Add a range of expressions together; (Lu 2.8.2; 2.8.4; 2.8.6)											Communication			
Subtract a range of expressions from each other. (Lu 2.8.2; 2.8.4; 2.8.6)											Problem solving			
											Independence			

good *average* *not so good*

Comments by the learner:			My plan of action:			My marks:		
I am very satisfied with the standard of my work.			<		*Date*:			

I am satisfied with the steady progress I have made.					Out of:			
I have worked hard, but my achievement is not satisfactory.					**Learner**:			
I did not give my best.			>					

Comments by parents:		Comments by teacher:
Parent signature: *Date*		Signature: *Date*:

Memorandum

CLASS ASSIGNMENT 1

- -12
- 5
- 31
- 18
- $–5$

- $4a$
- $10a + 9a$

- $-a + 8b – 38c$
- $–26a^2 – 16a – 12b – 10$

- $-7a + 3 – (6a – 9)$

$= -7a + 3 – 6a + 9$

$= -13a + 12$

- $18a^2 – 15 – (-7a^2 – 5a + 8)$

$= 18a^2 – 15 + -7a^2 + 5a – 8$

$= 25a^2 + 5a – 23$

- $-15x^2 – 7x + 20 – (-6x^2 + 76)$

$= -15x^2 – 7x + 20 + -6x^2 – 76$

$= -9x^2 – 7x – 56$

HOMEWORK ASSIGNMENT 1

1. $3p^2 – 17p – 25$

2. $8 – 7a^2 – (-5p^2 – 3p)$

$= 8 – 7a^2 + 5p^2 + 3p)$

$-2a^2 + 3p + 8$

3. $1 – (6a – 8y) = 1 – 6a + 8y$

4. $3a^3 + 6a – 7a – 5 – 16a^3 + 8a^2 – 34a – 16 – 15a$

$= -13a^3 + 8a^2 – 50a – 21$

5. $24p + 28y – 37a$

6. $-23a^2 + 3a + 5$

7. $12a^2 + 8a – 8$

8. $-6a + 6 – 5a^2$

Division in algebra

MATHEMATICS

Grade 8

THE NUMBER SYSTEM

(Natural and whole numbers)

Module 6

DIVISION IN ALGEBRA

CLASS ASSIGNMENT *1*

- Discover more and more about division in ALGEBRA!
- Write the following fraction in its simplest form in $\frac{45}{36}$ = ...
- Like common fractions, you can also write algebraic fractions in the simplest form.

What would the following be in its simplest form? $\frac{6\mathrm{a}^2b}{3\mathrm{ab}}$ = ...

Yes, it is actually like this: $\frac{6\times a\times a\times b}{3\times a\times b} = \frac{2\times a\times 1\times 1}{1\times 1\times 1}$

(Now you may cancel all the like terms above and below)(What remains above and below? Just write down the answer)

- There is a shortcut for terms with exponents: $\frac{m^5}{m^7} =$ ....

Are you able to identify the shortcut? Yes, 7 - 5 = 2. Therefore m^2 what remains below the line.

Answer: $\frac{1}{m^2}$

1. Now simplify the following:

1.1 $\frac{18m^5}{9\mathrm{m}^2}$ ....

1.2 $\frac{15p^4y^7}{3\mathrm{p}^7y^3}$ ....

1.3 $\frac{(4\mathrm{m}^2)^2}{2\mathrm{m}^6}$ ....

1.4 $\frac{4(2\mathrm{m}^2)^5}{2(m^2)^2}$ ....

1.5 $\frac{8\mathrm{a}^2(a^2)^2}{8\mathrm{a}^2}$ ....

1.6 $\frac{25a^2b^3c^4.4\mathrm{ab}^3c^2}{15a^3b^4c^5}$ ....

2.Remember: $\frac{1}{3}b(9\mathrm{a}) = \frac{1\times b}{3} \times \frac{9\mathrm{a}}{1} = \frac{b\times 9\mathrm{a}}{3} = \frac{9\mathrm{ab}}{3} = 3ab$

Therefore: $\frac{1}{3}$ means: **x 1 ÷ 3**

Now try to simplify the following: $^{1}/_{3}$ (4a - 6b)

2.1 Write each of the following in simplified form.

2.1.1: $\frac{1}{5}$ x a ....

2.1.2: $\frac{1}{5}(2a^2 - 15)$ ....

2.1.3: $(2a - 8b + 12c) \div 2$ ....

2.1.4: $\frac{6\mathrm{a}^2b^2c^3\text{- }15a^4b^6c^7+27b^9c^{10}}{3\mathrm{a}^2b^4c^3}$

2.1.5: $\frac{7\mathrm{m}^2\mathrm{pq}^9\text{- }49m^6n^7\text{- }35p^6q^{12}}{-7\mathrm{mn}^3q^4}$

HOMEWORK ASSIGNMENT *1*

1. Simplify:

1.1 $\frac{\text{- }56p^7q^7}{\text{-}8\mathrm{mn}^3q^4}$

1.2 $\frac{3\mathrm{a}^2\mathrm{bc}^4\text{- }36a^4b^7+24a^3b^2}{\text{-}3\mathrm{ab}^2c^2}$

1.3 ½ $(5a^2 - 25b)$

1.4 $\frac{(a^2b^2)^3.(\mathrm{ab}^2)^4}{a^2b^3}$

1.5 $\frac{3(4\mathrm{kp}^4)^2}{2\mathrm{k}^3p^2}$

2. If $P = 3ab^2 + 6a^2$ and $Q = 2ab$, calculate:

2.1 $2P - 3Q$

2.2 $\frac{P}{Q}$

2.3 $\frac{P+Q}{2\mathrm{Q}}$

3. Supposing that $5a^3b^2$ books cost $(-5ab + 15a^4b^7)$ rands, calculate the price of one book.

Assessment

Assessment of myself:		by myself:				Assessment by Teacher:								
I can…						1	2	3	4		Critical Outcomes	1	2	
express											Critical and			

fractions in their simplest form; (Lo 2.2; 1.6.2)											creative thinking			
express algebraic fractions in their simplest form; (Lo 2.2; 1.6.2; 2.8.3; 2.8.4; 2.8.5; 2.8.6)											Collaborating			
calculate the shortest path for terms with exponents. (Lo 2.2; 1.6.2; 1.6.3)											Organising en managing			
											Processing of information			
											Communication			
											Problem solving			
											Independence			

good *average* *not so good*

Comments by the learner:			My plan of action:			My marks:		
I am very satisfied with the standard of my work.			<		*Date*:			
I am satisfied with the steady progress I have made.					Out of:			
I have worked hard, but my achievement is not satisfactory.					**Learner**:			

I did not give my best.			>					

Comments by parents:		Comments by teacher:
Signature: *Date*:		Signature: *Date*:

Tutorial 2: (Algebra)

Total: 70

Question 1

A. Indicate whether the following statements are TRUE or FALSE and improve the wrong statements.

1. The base of $2x^3$ is $2x$.

2. The number of terms in the expression are THREE:$3(5a + 2) + 5b - 6$

3. $-10\frac{1}{2} < -10,499$

4. $-2^2 = -4$

5. The first prime number is 1.

6. $0 \div 6 = 0$

7. The 5 in 5^3 is known as the power.

8. $-3 > -6$

9. The square of van 8 is 64.

10. The coefficient of x in $4xy^2$ is 4.

11. $7(x - 2y) = 7x - 2y$

12. $2x^2 + 3x^2 = 5x^4$

13. $5 - 2 + (-2) = 1$

14. $\sqrt[3]{a^6b^{12}} = a^2b^4$

[14]

B. Complete the following:

1. The coefficient of a in $3ab$ is …

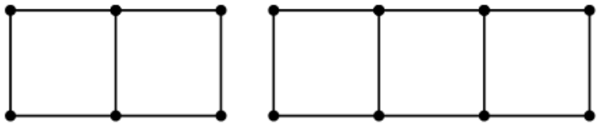
2. The exponent $5^{a + b}$ is ....

[2]

[16]

Question 2

Here matches are arranged to form squares.



1. Complete the following table. [4]

Number of squares	2	3	5	9
Number of matches				

2. How many matches are required to build 150 squares? [2]

3. How many squares can be built with 1 000 matches? [2]

[8]

Question 3

A. Supposing $a = 3$; $b = -2$ and $c = 6$, determine the value of:

1. $2a(3b + 5c)$ [3]

2. $\frac{a-b-c}{b-c}$ [4]

B. Addition and subtraction:

1. Add: $3a + 4b - 6c$; $-6b + 3c + 2a$; $5c - 3b$; $a - 2c$ [3]

2. Subtract $8x + 2y + 5$ from $10x - 2y + 4$ [3]

[13]

Question 4

Simplify each of the following:

1. $7a^2bc - 2ab^2c - 3ba^2c + acb^2$ [2]

2. $5a^2 \times 3a^3 - 3a^4 \times 2a$ [3]

3. $3(2a^2b^3)^2 \cdot (-ab)^3$ [4]

4. $-5ab^2 (3a^2 - 4b)$ [2]

5. $2a (7a + 4) - 3(a + 3a^2) - (-4a^2 - a)$ [4]

6. $\frac{(-3ab)^2(2a)^3}{4(ab)^2}$ [4]

7. $\frac{14a^4b^3}{6c^2} \times \frac{12c^4}{-7a^3b^2}$ [3]

[22]

Question 5

Supposing $A = a^2b + 5ab^2 - 6ab$ and $B = ab$, determine the following:

5.1 $\frac{A}{B}$ [3]

5.2 $\frac{3A}{B}$ [4]

5.3 $\frac{A+B}{B}$ [4]

[11]

TOTAL: 70

Algebra**Tutorial**

I demonstrate knowledge and understanding of:		Learning outcomes		0000	000	00	0
1.	different terms that can be distinguished in a polynomial;	2.4; 2.8.2; 2.9					
2.	identifying the coefficient of an unknown;	2.4; 2.9					
3.	identifying the constant in a polynomial;	2.4; 2.9					
4.	determining the degree of an expression;	2.4; 2.8.1; 2.9					
5.	arranging the expression in a descending order;	2.4; 2.9					
6.	the correct usage of signs (+ / -) in an expression;	2.4; 2.8.4 & .6					
7.	algebraic expression;	2.2; 2.4; 2.8.4					
8.	determining the formulas of flow diagrams and tables;	2.1 & .3; .4; .7					
9.	adding and subtracting of numbers;	2.4; 2.8.2 & .4					

10.	writing expressions in exponent form;	1.6.3; 2.4					
11.	solving expressions that have brackets;	2.4; 2.8.3 & .5					
12.	the correct usage of the order of calculations;	2.4; 2.8.5					
13.	substitution of unknowns with constant values;	2.4; 2.8.4;1.6.2 & .3					
14.	expressing fractions in their simplest form;	2.2; 2.4; 1.6.2					
15.	expressing algebraic fractions in their simplest form.	2.4; 1.6.2; 2.8.3					
16.							
17.							

The learner’s …	1	2	3	4
work is…	Not done.	Partially done.	Mostly complete.	Complete.
layout of the work is…	Not understandable.	Difficult to follow.	Sometimes easy to follow.	Easy to follow.
accuracy of calculations…	Are mathematically incorrect.	Contain major errors.	Contain minor errors.	Are correct.

	My BEST marks:				Comments by teacher:
Date:					
Out of:					
Learner:					
					Signature: *Date*:

Parent Signature: *Date*:

Test 2: (Algebra)

Total: 60

1. Simplify:

1.1: $17a^2bc - 21ab^2c - 3ba^2c + 4acb^2$ [2]

1.2: $5a^2 \times 3a^3 - 3a^4 \times 5a$ [3]

1.3: $7ab - (-4ab)$ [2]

1.4: $3(2a^2b^3)^3 \cdot (-a^2b^2)^3$ [3]

1.5: $-7ab^2(3a^2 - 5b)$ [2]

1.6: $5a(3a + 5) - 2(2a + 4a^2) - 2a(-3a)$ [4]

1.7: $\frac{(-2ab)^2(3a)^3}{4(ab^2)^2}$ [4]

[20]

2. If $a = 3$; $b = -2$ and $c = 6$, determine the values for:

2.1 $3a(2b + 4c)$ [3]

2.2 $\frac{a-b-c}{b+c}$ [4]

2.3 $\frac{3c}{a.b.c}$ [3]

2.4 $\frac{9a^5}{-3a^3}$ [3]

[13]

3. Given: $5a^2 - (m + n)a - 5p$

3.1 How many terms are there in the above expression? .... [1]

3.2 Write down the coefficient of p. .... [1]

3.3 Write down the exponent of p. .... [1]

3.4 What is the reciprocal of -5? .... [1]

[4]

4.

4.1 Determine by how much $7a + 5b + 9c$ is bigger than $2c + 3a - 7c$. [3]

4.2 Determine by how much $n + 3m + 3k$ is smaller than $-3k - 7m + 2n$ [3]

4.3 Subtract $x^2 - 2x + 4$ from $5c^2 + 6x - 9$ [3]

[9]

5. Write down an algebraic expression for each of the following.

5.1: 23 *m* reduced by the square of *n* .... [2]

5.2 Subtract the product of *m* and *n* from the difference between *m* and *n*. [2]

5.3 You have twenty coins. *y* of them are fifty-cent coins and the rest are ten-cent coins.

5.3.1 how many ten-cent coins do you have (in terms of *y*) [1]

5.3.2 What is the total value of your money? [2]

[7]

6. Complete the following tables and give a formula for each in the form *y* = ...

6.1 *y* = ... [3]

x	3	5	7	9	10	15	
y	9	25					

6.2 *y* = ... [4]

x	3	5	7	9	
y	11		23		38

[7]

7. Bonus question

Study the following pattern:

1 = 1 x 1

1 + 3 = 2 x 2

1 + 3 + 5 = 3 x 3

1 + 3 + 5 + 7 = 4 x 4

Now determine:

1 + 3 + 5 + 7 + 9 + ... + 21 + 23 + 25 + 27 + 29

2]

Enrichment exercise for the quick learner

1. If $p \quad q$ means $3p + q^2$, $(3 \quad 4) \quad 5$ will be equal to...

a) 60 b) 100 c) 87 d) 72 e) 91

2. In the multiplication shown, S and T are different digits between 1 and 9. The value of $S + T$ is ...

S 6

$\underline{\text{x } 2}\ T$

2 1 5 0

a) 13 b) 14 c) 15 d) 16 e) 17

3. How many digits are there in 5^8 ?

a) 2 b) 5 c) 6 d) 8 e) 40

4. The average of the numbers 0,1 ; 0,11 and 0,111 is ...

a) 0,041 b) 0,107 c) 0,11 d) 0,1111 e) 0,17

5. In a certain class ${}^{1}/_{3}$ of the pupils are girls. If 6 boys represent one quarter of the number of boys in the class, how many pupils are there in the class?

a) 18 b) 24 c) 27 d) 32 e) 36

6. If ½ - x = x - ${}^{1}/_{3}$, x will be equal to...

a) ${}^{1}/_{12}$ b) ${}^{1}/_{6}$ c) ${}^{5}/_{12}$d) ${}^{5}/_{6}$ e) ${}^{3}/_{12}$

7. Calculate the value of x if .....
Equation:

$$\sqrt[3]{4 + \sqrt{x}} = 2$$

a) 4 b) 2

Memorandum

CLASSWORK ASSIGNMENT 1

- $2m^3$
- **Equation:**

$$\frac{5\text{y}^4}{p^3}$$

- **Equation:**

$$\frac{8}{m^2}$$

- $\frac{4(32m^{10})}{2\text{m}^4}$ = $64m^6$

1.5 a^4

1.6 $\frac{20b^2c}{3}$

2. $\frac{4a}{3} - 2b$

2.1.1 $\frac{1a}{5}$

2.1.2 $\frac{2a^2}{5} - 3$

- $a - 4b + 6c$
- $\frac{2}{b^2} - 5a^2b^2c^4 + \frac{9b^5c^7}{a^2}$
- $\frac{-mpq^5}{n^3} + \frac{7m^5n^4}{q^4} + \frac{5p^6q^8}{mn^3}$

HOMEWORK ASSIGNMENT 1

1.1 $\frac{7p^7q^3}{mn^3}$

1.2 $\frac{-ac^2}{b} + \frac{12a^3b^5}{c^2} - \frac{8a^2}{c^2}$

1.3 $\frac{5a^2}{2} - \frac{25b}{2}$

$= \left(\frac{5a^2-25b}{2}\right)$

1.4 $\frac{a^6b^6.a^4b^8}{a^2b^3}$

$= a^8b^{11}$

1.5 $\frac{3\left(16k^2p^8\right)}{2k^3p^2}$

$= \frac{24p^6}{k}$

- $6ab^2 + 12a^2 - (6ab)$
- $\frac{3b}{2} + \frac{3a}{b}$
- $\frac{3ab^2+6a^2+2ab}{4ab}$

$= \frac{3b}{4} + \frac{3a}{2b} + \frac{1}{2}$

3. $\frac{-5ab+15a^4b^7}{5a^3b^2} = -\frac{1}{a^2b} + 3ab^5$

TUTORIAL

QUESTION 1

A.1. False: x

1. True
2. True
3. True
4. False: 2
5. True
6. False: 5 base
7. False
8. False

9. False: $4y^2$
10. False: $7x – 14y$
11. False: $5x^2$
12. True
13. True

B.1. $3b$

2. $a+b$

QUESTION 2

1. 7; 10; 16; 28

   1. (150 x 3) + 1 = 451
   2. (1 000 – 1) ÷ 3 = 333

QUESTION 3

A.1. 2(3)[3(–2) + 5(6)]

= 6[–6 + 30]

= 6[+24] = 144

2. $\frac{3-(-2)-(6)}{(-2)-(6)}$ or $\frac{3+2-6}{-2-6}$

= $\frac{-1}{-8}$ = $\frac{1}{8}$

B.1. $6a – 5b$

2. $2x – 4y – 1$

QUESTION 4

   1. $4a^2bc – ab^2c$
   2. $15a^5 – 6a^5 = 9a^5$
   3. $12a^4b^6(–a^3b^3)$

$= –12a^7b^9$

   1. $–15a^3b^2 + 20ab^3$
   2. $14a^2 + 8a – 3a – 9a^2 + 4a^2 + a$

$= 9a^2 + 6a$

6. $\frac{9\mathrm{a}^2b^2.8\mathrm{a}^3}{4\mathrm{a}^2b^2}$ $= 18a^3$

7. $\frac{14^2a^{a4}b^{b3}}{bc_1}$ x $\frac{12^2c^{4\,c2}}{-7a^3b^2}$

$= -4abc^2$

QUESTION 5

5.1 $\frac{a^2b+5\mathrm{ab}^2-6\mathrm{ab}}{\mathrm{ab}}$

$= a + 5b – 6$

5.2 $\frac{3\mathrm{a}^{2}b+15\mathrm{ab}^{2}-18\mathrm{ab}}{\mathrm{ab}}$

$= 3a + 15b – 18$

5.3 $\frac{a^{2}b+5\mathrm{ab}^{2}-5\mathrm{ab}}{\mathrm{ab}}$

$= a + 5b – 5$

TEST

- $4a^{2}bc – 17ab^{2}c$
- $15a^{5} – 15a^{5} = 0$
- $7ab + 4ab = 11ab$
- $3(8a^{6}b^{9}) (–a^{6}b^{6})$

$= –24a^{12}b^{15}$

- $–7a^{3}b^{2} + 35ab^{3}$
- $15a^{2} + 25a – 4a –8a^{2} + 6a^{2}$

$=13a^{2} + 21a$

- 3(3)[2(–2) + 4(6)]

= 9 [–4 + 24]

= 180

2.2 $\frac{3-(-2)-(6)}{(-2)+(6)}$

$= \frac{3+2-6}{-2+6}$

$= \frac{0}{4} = 0$

2.3 $\frac{3(6)}{(3)(-2)(6)}$

$= \frac{18}{-36}$

$= – \frac{1}{2}$

2.4 $\frac{9(3)^{5^{3^{2}}}}{-3(3)^{3}}$

$= \frac{9(9\overset{3}{)}}{-3}$

= –27

- 3
- –5
- 1
- **Equation:**

$$-\frac{1}{5}$$

- $7a + 5b + 9c – (2c + 3a – 7c)$

$= 7a + 5b + 9c – 2c + 3a – 7c$

$= 4a + 5b + 14c$

- $–3k – 7m + 2n – (n + 3m 3k)$

$= –3k – 7m + 2n – n + 3m 3k$

$= –6k – 10m + n$

- $5x^2 + 6x – 9 – (x^2 – 2x + 4)$

$= 5x^2 + 6x – 9 – x^2 – 2x + 4$

$= 4c^2 + 8x – 13$

- $23m – n^2$
- $(m – n) – mn$

- $20 – y$
- $50y – 10(20 – y)$

$= 50y – 200 + 10y$

$= 60y – 200$

- $y = x^2$

49; 81; 100; 225

- $y = 3x + 2$

17; 29; 18

7. $15^2 = 225$

ENRICHMENT EXERCISE

1. b
2. a
3. c
4. b
5. e
6. c
7. d

Lessons for Algebra I - Linear equations
This is an iMovie which is designed to help students and viewers to understand how to derive linear equations from few given points by calculating slopes and y-intercepts of straight lines. Then post their responses to given problems for further feedback.

Contex

This project is designed as an open educational source so that anyone learning online may may have access to the lessons. Traditional classroom students who miss classes and independent learners may have access to this project's lectures.

Objective

These lesson learners are able to develop linear equations from few given points by calculating slopes and y-intercepts.

Project Rationale/Purpose

The purpose of this project is to provide students who have missed classes a chance to catch up, to assist independent learners who might be in preparations of GED exams, and other online learners who wwould like to increase their math general knowledge. Additionally, this project is to support English Language Learners (ELL) who have difficulties keeping up with live lecture pace (Merryfield, 2003).

Why This project Was Put On YouTube?

YouTube is an interesting media that attracts a huge number of Internet surfers. It was created in 2005; however, it had the third largest Internet traffic behind Google and Yahoo in 2009. Thefefore, it is an ideal means for distance learning (Bonk, 2009).

Why Asynchronous Communication Was Chosen?

Asynchronous lectures are ideal in distance learn because they convenient for busy individuals who cannot afford to attend regular classes. In addition, they can access these lectures as many times as they like (Harism, 2000). Also asynchronous lectures are convenient for ELL because they cannot keep up with fast paced lectures, and they decode every word they hear in the lecture. At the time they complete processing the first few sentences of the lecture, the speaker may finish the speech or start talking about new materials (Merryfield, 2003). Many postsecondary and K-12 school institutions that implemented online courses and studies found that students learning online modestly perform better than students learning in classrooms (Means et al., 2010).

References:

Bonk, C. J. (2009). The world is open: How web technology is revolutionizing education. San Francisco, California: Jossey-Bass.

Harasim, L. (2000). Shift happens: online education as a new paradigm in learning. The Internet and Higher Education, 3(1-2), 41-61.

Merryfield, M. (2003). Like a veil- Cross-cultural experiential learning online. Contemporary Issues in Technology and Teacher Education, 3(2), 146-171.

Means, B., Toyama, Y., Murphy, R., Bakia, M., & Jones, K. (2010). Evaluation of evidence-based practices in online learning: A meta-analysis and review of online learning studies. *Structure*. Washington, DC: U.S. Department of Education. Retrieved from http://www2.ed.gov/rschstat/eval/tech/evidence-based practices/finalreport.pdf.

Relational Algebra

In this lecture, we will discuss the first formal languages for the relational models: Relational Algebra

In this lecture, we will discuss the first formal languages for the relational models: Relational Algebra

Relational Algebra

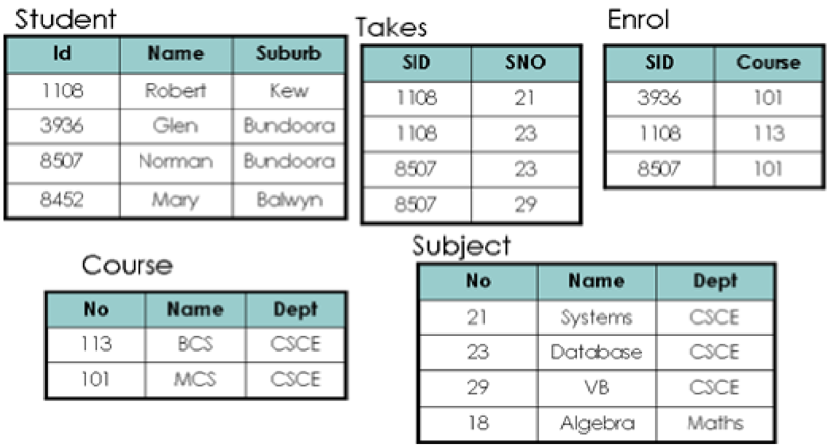
Relational Algebra (RA) can be viewed as a data manipulation language for relational model. It consists of several basic operations which is enable user to specify retrieval requests. RA is called a procedural language in which user need to specify how to retrieve the expected data.

Relational Algebra has the following components:

- Operands: Relations or Variables that represent relations
- Operators that map relations to relations
- Rules for combining operands and operators to relational algebra expression
- Rules for evaluating those expressions

Operations of relational algebra include the followings:

- Union, Intersect, Set Difference, Cartesian Product are operations based on set theory
- Select, Project, Join, Division are operations developed especially for relational databases.



Student

Id	Name	Suburb
1108	Robert	Kew
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

Takes

SID	SNO
1108	21
1108	23
8507	23
8507	29

Enrol

SID	Course
3936	101
1108	113
8507	101

Course

No	Name	Dept
113	BCS	CSCE
101	MCS	CSCE

Subject

No	Name	Dept
21	Systems	CSCE
23	Database	CSCE
29	VB	CSCE
18	Algebra	Maths

Example Database

Relational Algebra Operations from Set Theory

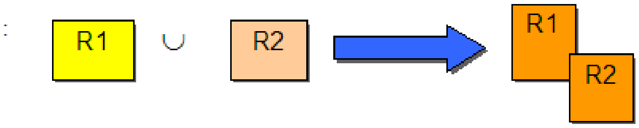
Definition: Two relations r(A1, A2, …, An) and s(B1, B2, …, Bn) are union compatible if they have the same degree n and dom(Ai) = dom(Bi) for 1 ≤ i ≤ n.

This mean two union compatible relations have the same number of attributes and each corresponding pair of attributes have the same domain

1. **UNION Operation**

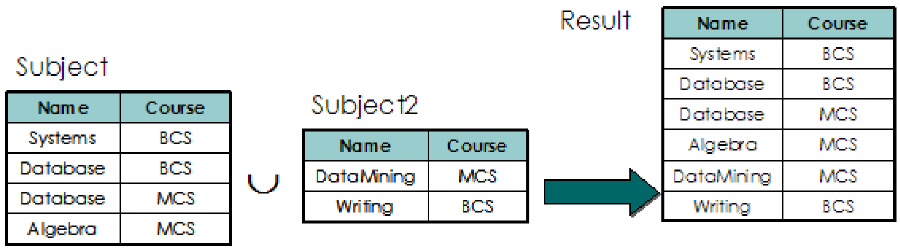
The UNION operation combines two union compatible relations into a single relation via set union of sets of tuples.

- Notation: $r1 \cup r2$



Union Operation Notation

- $r1 \cup r2 = \{t \mid t \in r1 \vee t \in r2\}$ where r1(R) and r2(R)
- Result size: $| r1 \cup r2 | \ \leq \ | r1 | + | r2 |$
- Result schema: R
- Producing the result of UNION
    - Make a copy of relation r1
    - For each tuple t in relation r2, check whether it is in the result or not. If it is not already in the result then place it there.
- Example:



Subject

Name	Course
Systems	BCS
Database	BCS
Database	MCS
Algebra	MCS

Subject2

Name	Course
DataMining	MCS
Writing	BCS

Result

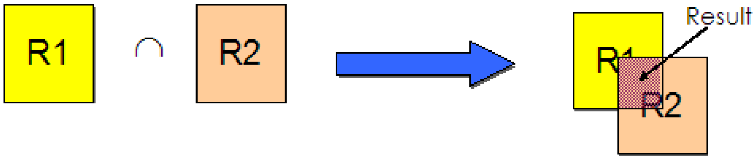
Name	Course
Systems	BCS
Database	BCS
Database	MCS
Algebra	MCS
DataMining	MCS
Writing	BCS

Union Operation Example

1. INTERSECTION Operation

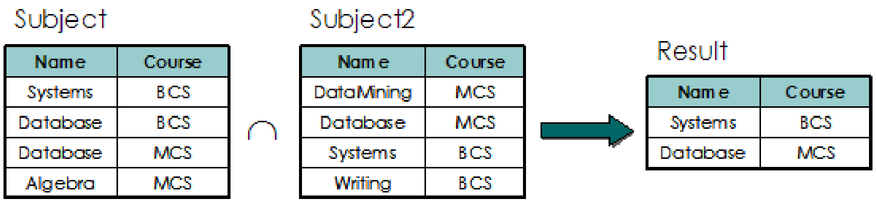
The INTERSECTION operation combines two union compatible relations into a single relation via set intersection of sets of tuples.

- Notation: $r1 \cap r2$



Intersection Operation Notation

- $r1 \cap r2 = \{t \mid t \in r1 \wedge t \in r2\}$ where r1(R) and r2(R)
- Result size: $| r1 \cap r2 | \leq \min(| r1 |, | r2 |)$
- Result schema: R
- Producing the result of INTERSECTION
  - Initially, result set is empty
  - For each tuple t in relation r1, if t is in the relation r2 then place t in the result set.
- Example



Subject

Name	Course
Systems	BCS
Database	BCS
Database	MCS
Algebra	MCS

Subject2

Name	Course
DataMining	MCS
Database	MCS
Systems	BCS
Writing	BCS

Result

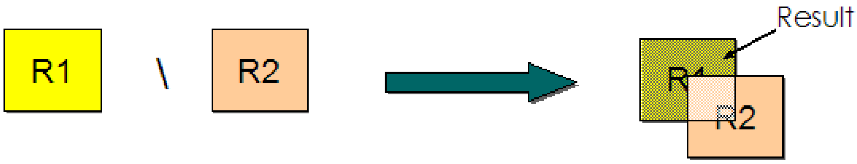
Name	Course
Systems	BCS
Database	MCS

Intersection Operation Example

1. SET DIFFERENCE Operation

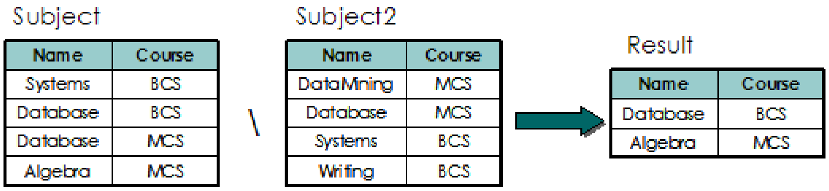
The DIFFERENCE operation finds the set of tuples that exist in one relation but do not occur in the other union compatible relation

- Notation: r1 \ r2



Difference Operation Notation

- $r1 = \{t \mid t \in r1 \wedge t \notin r2\}$ where r1(R) and r2(R)
- Result schema: R
- Producing the result of the DIFFERENCE operation
  - Initially, result set is empty
  - For each tuple in r1, check whether it appear in r2 or not. If it does not then place it in the result set. Otherwise, ignores it
- Example



Subject

Name	Course
Systems	BCS
Database	BCS
Database	MCS
Algebra	MCS

\

Subject2

Name	Course
DataMining	MCS
Database	MCS
Systems	BCS
Writing	BCS

Result

Name	Course
Database	BCS
Algebra	MCS

Difference Operation Example

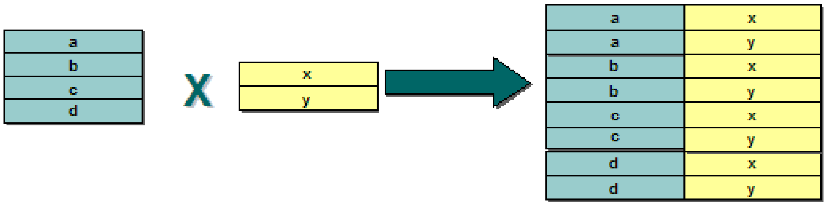
1. CARTESIAN PRODUCT Operation

The PRODUCT operation combines information from two relations pairwise on tuples.

- Notation: r x s
- $r \times s = \{(t1, t2) \mid t1 \in r \wedge t2 \in s\}$ where r(R) and s(S)
- Each tuple in the result contains all attributes in r and s, possibly with some fields renamed to avoid ambiguity. The result set contains all possible tuple that can be construct from one tuple in r and one tuple in s.
- Result schema: If we have R(A1, A2, …, An) and S(B1, B2, …, Bm) then the list of attributes in Result is (A1, A2, …, An, B1, B2, …, Bm)
- Result size: $|r \times s| = |r| * |s|$
- Producing the result of PRODUCT operation:

  - For each tuple in r, form new tuples by pair it with each tuple in s
  - Place all of these new tuples in the result set

- Example



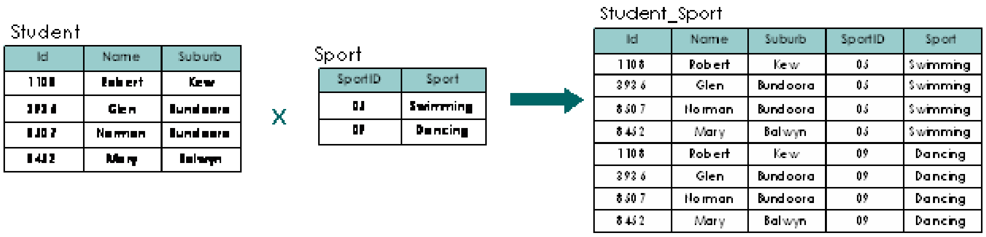
a
b
c
d

X

x
y

a	x
a	y
b	x
b	y
c	x
c	y
d	x
d	y

Catesian Product Operation Notation



Student

Id	Name	Suburb
1108	Robert	Kew
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

x

Sport

SportID	Sport
05	Swimming
09	Dancing

Student_Sport

Id	Name	Suburb	SportID	Sport
1108	Robert	Kew	05	Swimming
3936	Glen	Bundoora	05	Swimming
8507	Norman	Bundoora	05	Swimming
8452	Mary	Balwyn	05	Swimming
1108	Robert	Kew	09	Dancing
3936	Glen	Bundoora	09	Dancing
8507	Norman	Bundoora	09	Dancing
8452	Mary	Balwyn	09	Dancing

Catesian Product Operation Example

- Note: As we can notice, the CARTESIAN PRODUCT operation by itself is not a useful querying mechanism since the result size is large. However, it is an extremely important operation of relational algebra since it is the basic mechanism for combining information across relations. We will discuss about this topic in more detail in Query Processing lecture.

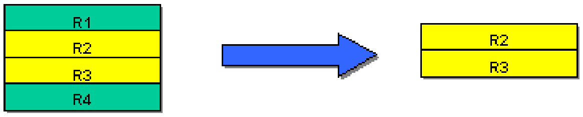
Other Relational Algebra Operations

1. **SELECT Operation**

The SELECT operation is an unary operation. It means the input of this operation is only one relation and it output is also a relation.

The SELECT operation returns a subset of the tuples from a relation that satisfies a selection condition. The SELECT operation can be viewed as a horizontal filter of the relation. It partitions the input relation into two sets of tuples: those tuples that satisfies the condition are select, those do not satisfy the condition are discarded.

- Notation: $\sigma_{\text{selection}-\text{condition}>}(r)$



Select Operation Notation

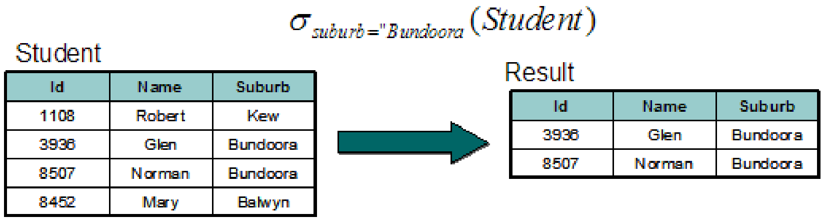
- $\sigma_F(r) = \{t \mid t \in r \wedge F(t)\}$ where r(R) and F is a boolean expression on attributes in R

The selection condition is made up of a number of clauses of the form

- <attribute name> <comparison op> <constant value> OR
- <attribute name 1> <comparison op> <attribute name 2>

In the clause, the comparison operations could be one of the following: ≤, ≥, ≠, =, >, < . Clauses are connected by Boolean operators : and, or , not

- Result size $\mid \sigma_F(r) \mid \leq \ \mid r \mid$
- Result schema: R
- Producing the result of the SELECT operation
  - Selection condition F is evaluate for each tuple in r, with the attribute variables in F set to their values in the tuples
  - Any tuple t that F(t) = true is placed in the result set
  - Other tuples are not include in the result.
- Example: Retrieve the Id, Name, Suburb of students who live in Bundoora



$$\sigma_{suburb="Bundoora}(Student)$$

Student

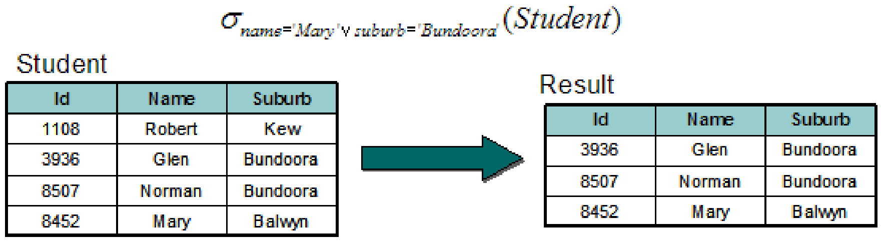
Id	Name	Suburb
1108	Robert	Kew
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

Result

Id	Name	Suburb
3936	Glen	Bundoora
8507	Norman	Bundoora

Select Operation Example - 1

- Example: Retrieve the Id, Name, Suburd of student who’s name is Mary or students who live in Bundoora



$\sigma_{name='Mary' \vee suburb='Bundoora'}(Student)$

Student

Id	Name	Suburb
1108	Robert	Kew
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

Result

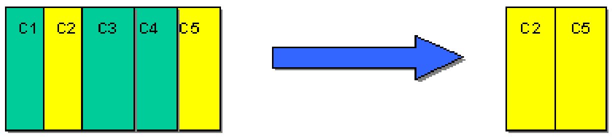
Id	Name	Suburb
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

Select Operation Example - 2

1. PROJECT Operation

The PROJECT operation is another unary operation. This operation returns a set of tuples containing a subset of the attributes in the original relation. Thus, as we state that the SELECT operation selects some rows and discards the others. The PROJECT operation, on the other hand, selects some columns of the relation and discards the other column. The PROJECT operation can be viewed as the vertical filter of the relation.

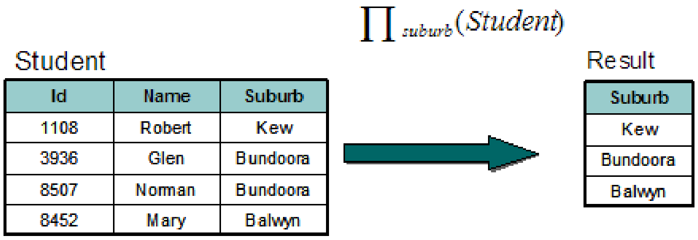
- Notation: $\pi_{\text{attribute}-\text{list}}(r)$



Project Operation Notation

- $\pi_X(r) = \{t[X] \mid t \in r\}$ where r(R)
- Result size: $|\pi_X(r)| \leq |r|$
- Result schema: R’(X)
- Producing the result of PROJECT operation
    - Take each tuple in the original relation, extract the values of the specified attributes
    - Form new tuple from these values and place the new tuple in the result if it is not already there. This steps includes the duplicate removal phase, this makes the result of PROJECT operation a relation

- Example: Retrieve the suburbs that are stored in database



$\prod_{suburb}(Student)$

Student

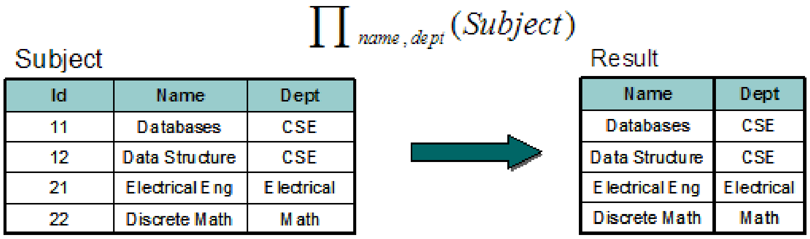
Id	Name	Suburb
1108	Robert	Kew
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

Result

Suburb
Kew
Bundoora
Balwyn

Project Operation Example - 1

- Retrieve the name of the subjects and department which is responsible for the subject



$\prod_{name,\, dept}(Subject)$

Subject

Id	Name	Dept
11	Databases	CSE
12	Data Structure	CSE
21	Electrical Eng	Electrical
22	Discrete Math	Math

Result

Name	Dept
Databases	CSE
Data Structure	CSE
Electrical Eng	Electrical
Discrete Math	Math

Project Operation Example - 2

1. **JOIN Operation**

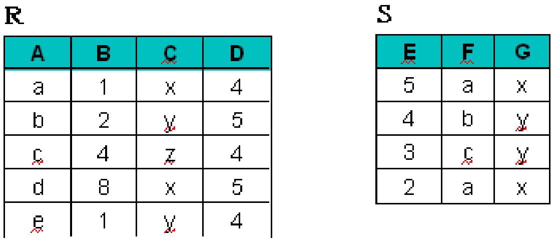
The JOIN operation is used to combine related tuples from two relations into a single tuple. The Theta-JOIN is a specialized product containing only pairs that match on a supplied condition called join-condition.

- Notation: $r \quad {}_{\text{join}-\text{condition}}\, s$
- $r \quad {}_{C}\, s = \{(t1\,,t2) \mid t1 \in r \wedge t2 \in s \wedge C(t1\,,t2)\}$where r(R) , s(S)
- Similar to PRODUCT, each tuple in the result of JOIN operation contains all attributes from two original relations. However, in this operation one tuple in R and one tuple in S can be combined together to form a tuple in the result if the combination satisfies the join condition.
- Join condition is of the form:

<condition> AND <condition> AND …AND <condition>

where <condition> is a comparision between one attribute in R and one attribute in S, provided that these two attributes have the same domain.

- Result size: $|\, r \quad {}_{C}\, s \,| \leq |\, r \,| * |\, s \,|$
- Result schema: If we have R(A1, A2, …, An) and S(B1, B2, …, Bm) then the list of attributes in Result is (A1, A2, …, An, B1, B2, …, Bm)
- Producing the result of JOIN operation:
    - For each tuple in r, form new tuples by pair it with each tuple in s
    - If the new tuple satisfies the specified condition, then place it in the result set.
- Example:



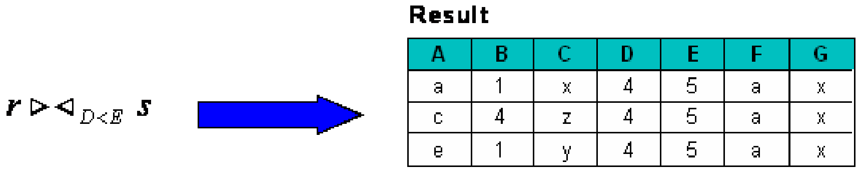
R

A	B	C	D
a	1	x	4
b	2	y	5
c	4	z	4
d	8	x	5
e	1	y	4

S

E	F	G
5	a	x
4	b	y
3	c	y
2	a	x

Join Operation Example - 1

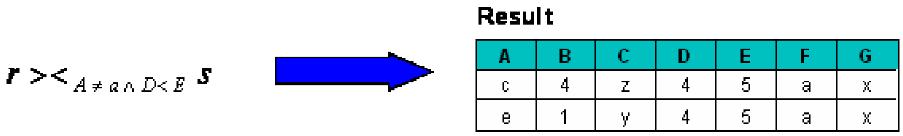


$r \rhd\!\lhd_{D<E}\, s$

Result

A	B	C	D	E	F	G
a	1	x	4	5	a	x
c	4	z	4	5	a	x
e	1	y	4	5	a	x

Join Operation Example - 2



$r >\!<_{A \neq a \wedge D<E}\, s$

Result

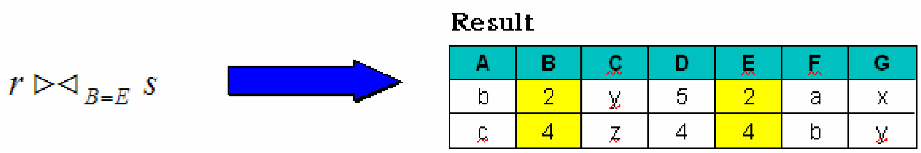
A	B	C	D	E	F	G
c	4	z	4	5	a	x
e	1	y	4	5	a	x

Join Operation Example - 3

Variations of JOIN

EQUI-JOIN: A JOIN where the only comparision operator used in the join condition is “=” is called EQUI-JOIN. The result of Equi Join always has one or more pairs of attributes that have identical values in every tuple.

Example:



$r \bowtie_{B=E} s$

Result

A	B	C	D	E	F	G
b	2	y	5	2	a	x
c	4	z	4	4	b	y

Natural Join Operation Example - 1

NATURAL JOIN: The Natural Join operation is a specialised product where the result tuple contains only pairs of tuples that match on their common attributes with one of each pair of common attributes is eliminated. The standard definition of Natural Join requires that the two join attributes have the same name. Therefore, we can see that Natural Join is created to get rid of the duplicate columns in an Equi Join.

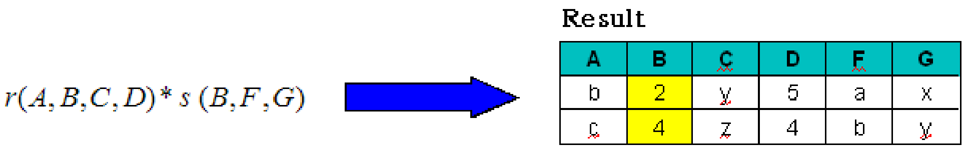
- Notation: r * s
- Natural Join can be defined using other operation $r * s = \pi_{R \cup S}(\sigma_{\text{condition}}(r \times s))$

where r(R) and s(S) and condition is boolean expression (A1 = B1) AND (A2 = B2) AND … AND (Ak = Bk) with Ai is the attribute in r , Bi is the attribute in s and (Ai, Bi) is a pair of common attributes.

Producing the result of Natural Join

- For each tuple in relation r, compare common attributes with those in each tuple of s
- If two tuples match in their common attributes then combine tuples, remove duplicate attributes and add to the result.

- Example: From the example of Equi Join, assume that the attribute list in s now is (E, F,B) instead of (E, F, G) then we can have the expression r * s .

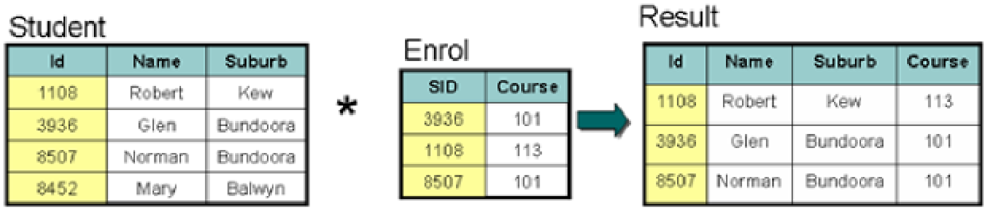


$r(A,B,C,D) * s(B,F,G)$

Result

A	B	C	D	F	G
b	2	y	5	a	x
c	4	z	4	b	y

- Retrieve the information of student who enrols in at least one course.



Student

Id	Name	Suburb
1108	Robert	Kew
3936	Glen	Bundoora
8507	Norman	Bundoora
8452	Mary	Balwyn

*

Enrol

SID	Course
3936	101
1108	113
8507	101

Result

Id	Name	Suburb	Course
1108	Robert	Kew	113
3936	Glen	Bundoora	101
8507	Norman	Bundoora	101

Natural Join Operation Example - 2

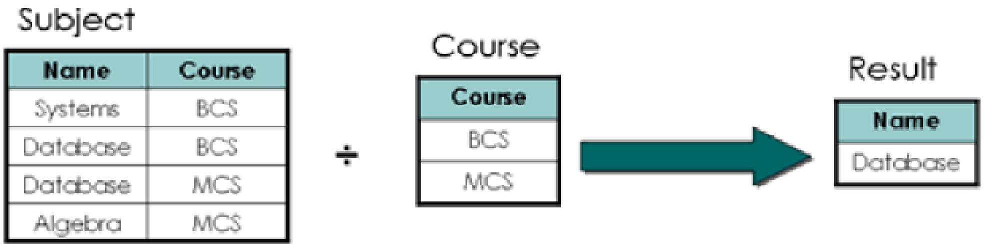
1. **DIVISION Operation**

The Divition Operation is defined on two relation r(U1) and s(U2) where U2 is the subset of U1 and s is not an empty relation: $r \div s = \{t \mid t \in r(U_1 - U_2) \wedge \text{satisfy}\}$ where $\text{satisfy} = \forall t_s \in s(\exists t_r \in r(t_r[U_2] = t_s \wedge t_r[U_1 - U_2] = t))$

This means that for a tuple t to appear in the result of Division, the values in t must appear in r in combination with every tuple in s.

The Division is very useful for a special kind of query such as “ Retrieve the name of the student who enrolls in all course teach by Professor Ba”

- Producing the result of the Division operation
    - Consider each subset of tuple in r that match on t[U1 – U2]
    - For this subset of tuples, take the values t[U2] from each. If this covers all tuples in s then add t[U1 – U2] in the result.
- Example: Retrieve the name of subject that is taught in all courses



Subject

Name	Course
Systems	BCS
Database	BCS
Database	MCS
Algebra	MCS

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Course

Course
BCS
MCS

Result

Name
Database

Division Operation Example

Data Manipulation with Relational Algebra Expression

Sample Database

In this session, we use the COMPANY database in the examples for illustrating the use of Relational Algebra for answering several queries.

The relational database schema for the COMPANY database is specified as below

EMPLOYEE (EID, Name, Bdate, Address, Salary, DeptId)

DEPARTMENT(DeptId, Dname, Office, Mng-EID)

PROJECT(Code, Name, Budget, DeptId)

JOIN(EID, PCode, StartDate)

EMP-DEPENDENT(EID, Dependent-Name, Bdate, Relationship)

Sample Queries

Query 1: Find all employees whose salary is greater than 30.000

$\sigma_{\text{salary}>30000}(\text{EMPLOYEE})$

Query 2: Find the name, address of employees who works for department number 1

$\pi_{\text{Name,Address}}(\sigma_{\text{DeptId}=1}(\text{EMPLOYEE}))$

Query 3: Find the name of the department that employee John Smith works for.

$\pi_{\text{Dname}}(\sigma_{\text{Name}=\text{'JohnSmith'}}(\text{EMPLOYEE} * \text{DEPARTMENT}))$

Query 4: Find the EID, start date of the employees who works for project number P1 or project number P2

$$\pi_{\mathrm{EID,StartDate}}(\sigma_{\mathrm{PCode=1 \vee PCode=2}}(\mathrm{JOIN}))$$

Query 5: Find the name, relationship of all the dependents of employees who works for Department Human Resource

$$\pi_{\mathrm{Dependent-Name,Relationship}}(\sigma_{\mathrm{DName='HumanResource'}}$$

$$(\mathrm{EMP - DEPENDENT} * \mathrm{EMPLOYEE} * \mathrm{DEPARTMENT}))$$

Query 6: Finds the name of the employees who join in every project.

$$\pi_{\mathrm{Name}}(\mathrm{EMPLOYEE} * (\pi_{\mathrm{EID,PCode}}(\mathrm{JOIN}) \div \pi_{\mathrm{Code}}(\mathrm{PROJECT})))$$

Query 7: Finds the names of the employees who have no dependent

$$\pi_{\mathrm{Name}}(\mathrm{EMPLOYEE} * (\pi_{\mathrm{EID}}(\mathrm{EMPLOYEE})_{\mathrm{EID}}(\mathrm{EMP - DEPENDENT})))$$

Query 8: Find the name of employees who works for both project number 1 and project number 2

$$\pi_{\mathrm{Name}}(\mathrm{EMPLOYEE} * (\pi_{\mathrm{EID}}(\sigma_{\mathrm{PCode=1}}\mathrm{JOIN}) \cap \pi_{\mathrm{EID}}(\sigma_{\mathrm{PCode=2}}\mathrm{JOIN})))$$

Query 9: Find the name of the manager who has at least one dependent

$$\pi_{\mathrm{Name}}(\mathrm{EMPLOYEE} * (\pi_{\mathrm{EID}}(\mathrm{EMP - DEPENDENT} \quad {}_{\mathrm{EID=Mng-EID}}\mathrm{DEPARTMENT})))$$